Online Appendix for
"Wage Volatility and Changing Patterns of Labor Supply"

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Appendix I. Data

We use data from Panel Study of Income Dynamics (PSID) in 1968 through 2011 to document how wages, hours worked, consumption, and wealth by skill level have evolved. Since the survey is conducted biennially beginning 1997, we exploit a total of 37 surveys from PSID. We also exploit data from Current Population Survey (CPS) March Supplements to document the trends in the skill premium and the skilled-unskilled hours differential for comparison purposes.

PSID: We begin with the core Survey Research Center (SRC) sample, which represents the U.S. population in 1968. We restrict our sample to male heads of household who participated in the labor market last year (i.e., worked at least 260 hours).We only include men aged between 25 and 59 with non-missing information about their educational attainment, total annual work hours, and labor income in the past calendar year. Hourly wage is obtained by dividing annual labor income by annual work hours. Extreme outliers are excluded from our sample by dropping those whose reported annual hours worked in the past year are more than 5840 hours, or whose hourly wage is less than half the federal minimum wage. Self-employed men are also removed because it is difficult to distinguish between labor and capital shares out of their income. The resulting sample is an unbalanced panel. We also extract information on household wealth and consumption for this sample. Household wealth is defined by net worth based on farm and business assets, checking/savings accounts, stocks, IRA/private annuities, net worth of vehicles, home equity, net worth of other real estate, other assets, and other debts. These variables are available for years 1984, 1989, 1994, 1999, 2001, 2003, and 2007. Household consumption is defined by the sum of food at home and food away from home, available for all survey years except for 1973, 1988, and 1989. This food expenditure is divided by the number of adult equivalents, where adult equivalent is defined by $(\text{number of adults} + 0.7(\text{number of children}))^{0.7}$ according to the census equivalence scale. We use the consumption per adult equivalent for our analysis.

CPS: We apply the same sample selection criteria to data from the CPS. We include men aged between 25 and 59 with reported educational attainment. We obtain annual hours worked by multiplying previous calendar year's weeks worked by usual hours worked per week. In surveys before 1976, usual hours worked per week in the past year are not available and weeks worked in the past year are coded in intervals. Therefore, we impute both variables for previous surveys using the average weeks worked and the average usual hours worked per week in the same education and weeks worked interval cells in the 1976 survey. Annual earnings in the CPS are income from wages and salaries. We multiply top-coded earnings by 1.5, following Katz and Murphy (1992). Hourly wage is annual earnings divided by annual hours worked. As with the PSID, we exclude those who worked less than 260 hours and more than 5840 hours in the past year, who earned less than half the federal minimum wage per hour, or who is currently self-employed.

Appendix II. Estimation of Wage Processes

As described in section 2.1, following Heathcote et al. (2010), we model the log wage residual y_{it}^e as the sum of persistent and transitory shocks with time-varying variances:

$$y_{it}^e = \mu_{it}^e + \upsilon_{it}^e + \theta_{it}^e$$

where μ_{it}^e is a persistent component, $\upsilon_{it}^e \sim (0, \lambda_t^{e,\upsilon})$ is a transitory component, and $\theta_{it}^e \sim (0, \lambda^\theta)$ is measurement error. The persistent component μ_{it}^e is assumed to follow an AR(1) process:

$$\mu_{it}^e = \rho^e \mu_{it-1}^e + \eta_{it}^e$$

where ρ^e is the persistence and $\eta_{it}^e \sim (0, \lambda_t^{e,\eta})$ is a persistent wage shock whose variance $\lambda_t^{e,\eta}$ varies over time. The initial value of the persistent component is drawn from a skill-specific distribution: $\mu_0^e \sim (0, \lambda^{e,\mu})$. We assume that all four variables, υ_{it}^e, θ_{it}^e, η_{it}^e, and μ_0^e are orthogonal and i.i.d. across individuals.

As we mention in 2.1, we take the estimate of 0.02 from French (2004) for the variance, λ^θ, of measurement error. Then, we estimate a parameter vector Φ^e, which includes two time-invariant parameters ρ^e and $\lambda^{e,\mu}$ and a set of time-varying parameters $\{\lambda_t^{e,\eta}, \lambda_t^{e,\upsilon}\}_{t=1967}^{2006}$ for each skill group $e \in \{s, u\}$. Since the PSID are available biennially beginning in 1997, we do not have empirical moments for transitory shocks for years 1997 and 1999. In order to resolve this issue, we assume that the cross-sectional variance of log residual wages in these missing years is the average of that in the previous year and in the subsequent year, and identify the variances of transitory wage shock for missing years as is done in Heathcote et al. (2010).

For each sample year t, we construct 10-year adjacent age cells from ages 29 to 54 such that, for instance, the age group 29 consists of those aged 25 to 34 years. We then compute the empirical autocovariance, $\widehat{g}_{a,t,n}^e$, of all possible orders for each age/year (a, t) cell in our PSID sample using log wage residuals $\widehat{y}_{it}^e$ from the first-stage regressions:

$$\widehat{g}_{a,t,n}^e = \frac{1}{I_{a,t,n}^e} \sum_{i=1}^{I_{a,t,n}^e} \widehat{y}_{it}^e\big|_{a_{it}=a} \cdot \widehat{y}_{i,t+n}^e\big|_{a_{it}=a}, \quad n \geqslant 0,$$

where $I_{a,t,n}^e$ is the number of observations for nth order autocovariance for age/year (a, t) cell in skill group e. We then pick the parameters $\widehat{\Phi}^e$ that minimize the equally weighted distance between this empirical autocovariance matrix and its theoretical counterpart:

$$\widehat{\Phi}^e = \arg\min_{\Phi^e} \left[\widehat{G}^e - G^e(\Phi^e)\right]' I \left[\widehat{G}^e - G^e(\Phi^e)\right],$$

where $\widehat{G}^e$ is a stacked vector of empirical autocovariances as well as cross-sectional variances for missing years, $G^e(\Phi^e)$ is the theoretical counterpart, and I is an identity matrix. Our estimation strategy is closest to that in Heathcote et al. (2010). As a robustness check, we estimate the variances of persistent and transitory wage shocks of the whole sample as Heathcote et al. (2010) do and confirm that our estimates are consistent with theirs.

Appendix III. Algorithm

A set of parameters β^s, β^u, ψ^s, and ψ^u is calibrated to match targets in the initial steady state. We allow the parameters $\mathbf{\Lambda}_t = \{\chi_t, \pi_t, \lambda_t^{s,\eta}, \lambda_t^{s,\upsilon}, \lambda_t^{u,\eta}, \lambda_t^{u,\upsilon}\}$, governing the skill premium, the population share of the skilled, and persistent and transitory idiosyncratic productivity, to vary over time.

Solving for the steady state

Under a set of parameters for the steady state (including $\mathbf{\Lambda}_*$),

1. Guess price r. Given this guess, compute w^u and w^s (given $\mathbf{\Lambda}_*$).
2. Solve the value function and get $h(e, a, \mu^e, \upsilon^e), a'(e, a, \mu^e, \upsilon^e), c(e, a, \mu^e, \upsilon^e)$.
3. Generate a sample of population N over $(e, a, \mu^e, \upsilon^e)$ space. That is, $N_S = \pi_* N$ sample over (a, μ^s, υ^s) and $N_U = (1 - \pi_*)N$ sample over (a, μ^u, υ^u).
4. Compute aggregate statistics:
$$K = \sum_e \int a dG(e, a, \mu^e, \upsilon^e)$$
$$S = \int h(s, a, \mu^s, \upsilon^s) \ \exp(\mu^s + \upsilon^s) \ dG(s, a, \mu^s, \upsilon^s)$$
$$U = \int h(u, a, \mu^u, \upsilon^u) \ \exp(\mu^u + \upsilon^u) \ dG(u, a, \mu^u, \upsilon^u)$$
$$H = \left\{ \chi U^\phi + (1 - \chi) S^\phi \right\}^{\frac{1}{\phi}}$$
   And define
$$\tilde{r} = z\alpha \left(K/H \right)^{\alpha - 1} - \delta$$
5. Check if $|r - \tilde{r}| < \epsilon$. If not, update r and go back to step 1.

Solving transition economy (Changing parameters over time)

The economy was originally at the initial steady state $(*)$. There is a gradual change for $\mathbf{\Lambda}$ from $\mathbf{\Lambda}_*$ to $\mathbf{\Lambda}_{**}$ for periods $t = t_1, \cdots, t_\tau$, and then the economy converges to the new steady state $(**)$ at $t_T (> t_\tau)$.

1. A sequence of parameters is given: $\{\omega_t, \lambda_t^{s,\eta}, \lambda_t^{s,\upsilon}, \lambda_t^{u,\eta}, \lambda_t^{u,\upsilon}\}_{t=t_1}^{t_\tau}$.

2. Solve two (initial and final) steady states and find a set of parameters $\{\chi_t, r_t\}_{t=t_0=*}$, $\{\chi_t, r_t\}_{t=t_T=**}$, where χ_t is adjusted so that the model matches the observed skill premium (sp_t) in each steady state. Record the stationary distribution $G_*(e, a, \mu^e, \upsilon^e)$ and the value function $V_{**}^e(a, \mu^e, \upsilon^e)$.

3. Guess the sequences of $\{\chi_t, r_t\}_{t=t_1}^{t_T-1}$. Then, the other prices are given by

$$w_t^s = w^s(\chi_t, r_t; \alpha, \delta, \phi)$$
$$w_t^u = w^u(\chi_t, r_t; \alpha, \delta, \phi)$$

$$w_t^u = z(1-\alpha)\chi_t \left(\frac{r_t+\delta}{z\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left(\chi_t + (1-\chi_t)\left(\frac{\chi_t}{1-\chi_t}\cdot\omega_t\right)^{\frac{\phi}{\phi-1}}\right)^{\frac{1-\phi}{\phi}}$$
$$w_t^s = w_t^u \cdot \omega_t$$

4. Solve for workers decisions backwards and get the decision rules during the transition periods:

   (a) Using the value function at the new steady state: $V_{**}^e(a, \mu^e, \upsilon^e; \mathbf{\Lambda}_{**})$,

   (b) In each period during the transition ($t = t_1, \cdots, t_{T-1}$), we solve

   $$V_t^e(a_t, \mu_t^e, \upsilon_t^e) = \max_{c_t, a_{t+1}, h_t} \{u(c_t, h_t) + \beta^e \gamma \mathbb{E}_t V_{t+1}^e(a_{t+1}, \mu_{t+1}^e, \upsilon_{t+1}^e)\}$$

   subject to constraints with $p_t = \{r_t, w_t^u, w_t^s\}$ and $\mathbf{\Lambda}_t = \{\chi_t, \pi_t, \lambda_t^{s,\eta}, \lambda_t^{s,\upsilon}, \lambda_t^{u,\eta}, \lambda_t^{u,\upsilon}\}$ for $t = t_1, \cdots, t_{T-1}$.

   - Solve the problem above backwards from $t = t_{T-1}, \cdots, t_1$. In each period, we obtain the decision rules: $c_t(e, a_t, \mu_t^e, \upsilon_t^e)$, $h_t(e, a_t, \mu_t^e, \upsilon_t^e)$, $a_t'(e, a_t, \mu_t^e, \upsilon_t^e)$ and the value function, $V_t^e(a_t, \mu_t^e, \upsilon_t^e)$.

5. Given a distribution at t, $G_t(e, a_t, \mu_t^e, \upsilon_t^e)$, simulate $G_{t+1}(\cdot)$ by applying $c_t(\cdot), h_t(\cdot)$, and $a_t'(\cdot)$. Start from $t = t_1$. Note that we already know $G_{t_1}(\cdot)$ from the distribution of the initial steady state $G_*(\cdot)$

   (a) First calculate the aggregate statistics:

   $$K_t = \sum_e \int a_t dG_t(e, a_t, \mu_t^e, \upsilon_t^e)$$
   $$S_t = \int h_t(s, a_t, \mu_t^s, \upsilon_t^s)\exp(\mu_t^s + \upsilon_t^s) dG_t(s, a_t, \mu_t^s, \upsilon_t^s)$$
   $$U_t = \int h_t(u, a_t, \mu_t^u, \upsilon_t^u)\exp(\mu_t^u + \upsilon_t^u) dG_t(u, a_t, \mu_t^u, \upsilon_t^u).$$

(b) Using these aggregate statistics, compute

- an updated share $\tilde{\chi}_t$ (which matches the observed skill premium (sp_t)) such that

$$\tilde{\chi}_t = \frac{(S_t/U_t)^{\phi-1}}{\omega_t + (S_t/U_t)^{\phi-1}},$$

where $\omega_t = w_t^s/w_t^u$.
Note that $sp_t = [w_t^s \times E(\exp(\mu_t^s + \upsilon_t^s))]/[w_t^u \times E(\exp(\mu_t^u + \upsilon_t^u))]$. Hence, $\omega_t = sp_t \times E(\exp(\mu_t^u + \upsilon_t^u))/E(\exp(\mu_t^s + \upsilon_t^s))$.

- and an implied price $\tilde{r}_t$ such that

$$\tilde{r}_t = z_t\alpha\left(K_t/\tilde{H}_t\right)^{\alpha-1} - \delta,$$

where

$$\tilde{H}_t = \left\{\tilde{\chi}_t U_t^{\phi} + (1-\tilde{\chi}_t)S_t^{\phi}\right\}^{\frac{1}{\phi}},$$

(c) Get an updated distribution G_{t+1} and go back to (5a) until we get G_{t_T}.

6. Check if the guessed sequences $\{\chi_t, r_t\}_{t=*}^{**}$ are close enough to the model-implied sequences of $\{\tilde{\chi}_t, \tilde{r}_t\}_{t=*}^{**}$ from (5b). If not, update the guesses and go back to step 3.

References

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